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application notes

Averaging Time of Measurements

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Averaging Time of Measurements

Time averaging together with some sort of weighting are the two most important processes in the data reduction which is necessary in almost any measurement of a signal. For signals involving only high frequencies the time averaging normally causes no problems, but for low frequency signals or single pulses it is more important to know the influence of the averaging process and of a finite averaging time. The main purpose of this paper is to compare two simple types of averaging and find out how to choose the parameters to get most equal results.

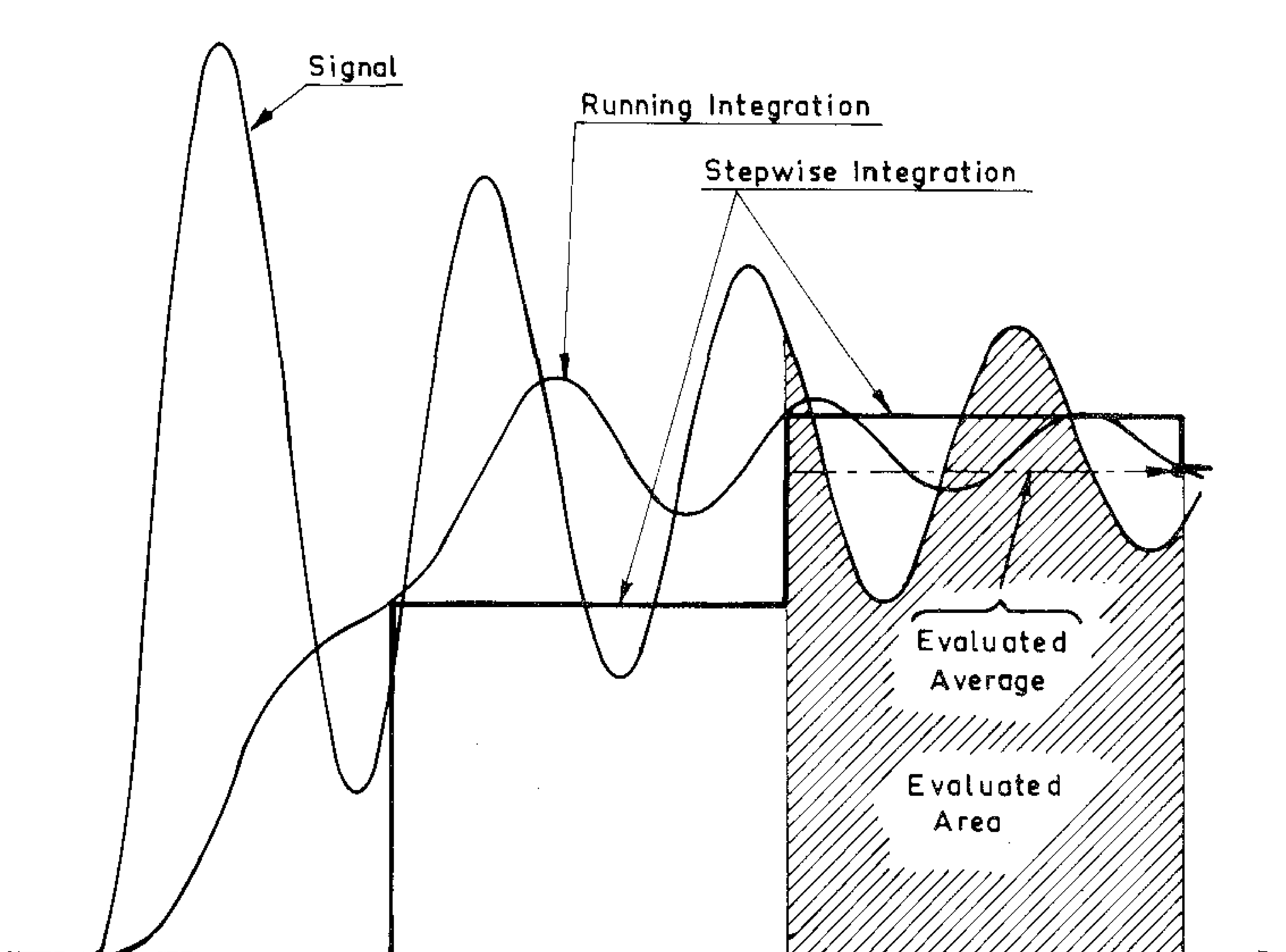
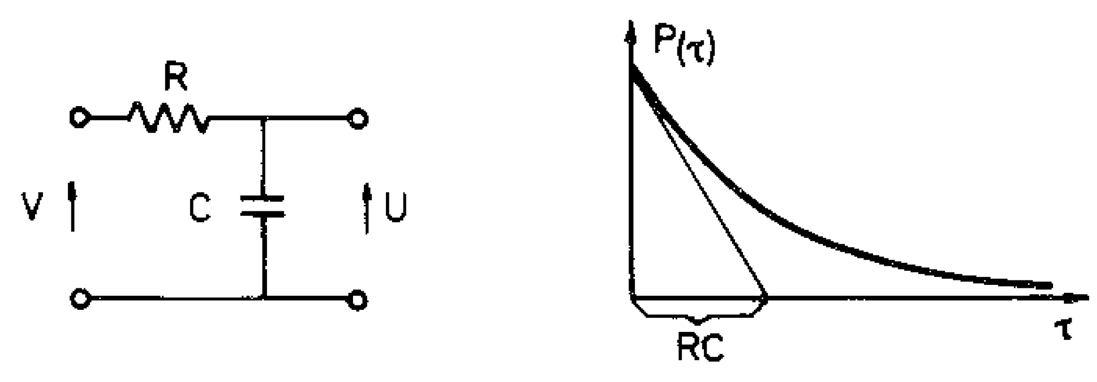


Fig. 1. True Integration of a Signal

Mathematically the most simple averaging is a true integration of the signal over a finite time T, the result being divided by T. Ref. fig.1. If one wants to follow the signal later on, the integration interval could be moved continuously giving always an averaging of the last T seconds of the signal. This is called running integration in the following. However, this requires a complete memory of the last T seconds of the signal to be able to throw away the values just running out of the integration period. In most cases it is therefore preferred to do the integration stepwise in fixed time intervals, thereby obtaining sample values of the curve that the running integration would give. This is called stepwise integration in the following.



$V = RC\frac{dU}{dt} + U$ $\qquad P(\tau) = \exp(-\tau/RC)$

$U(t) = \int_0^{\infty} V(t-\tau)\cdot P(\tau)\,d\tau$

Fig. 2. Some formulas for RC-averaging.

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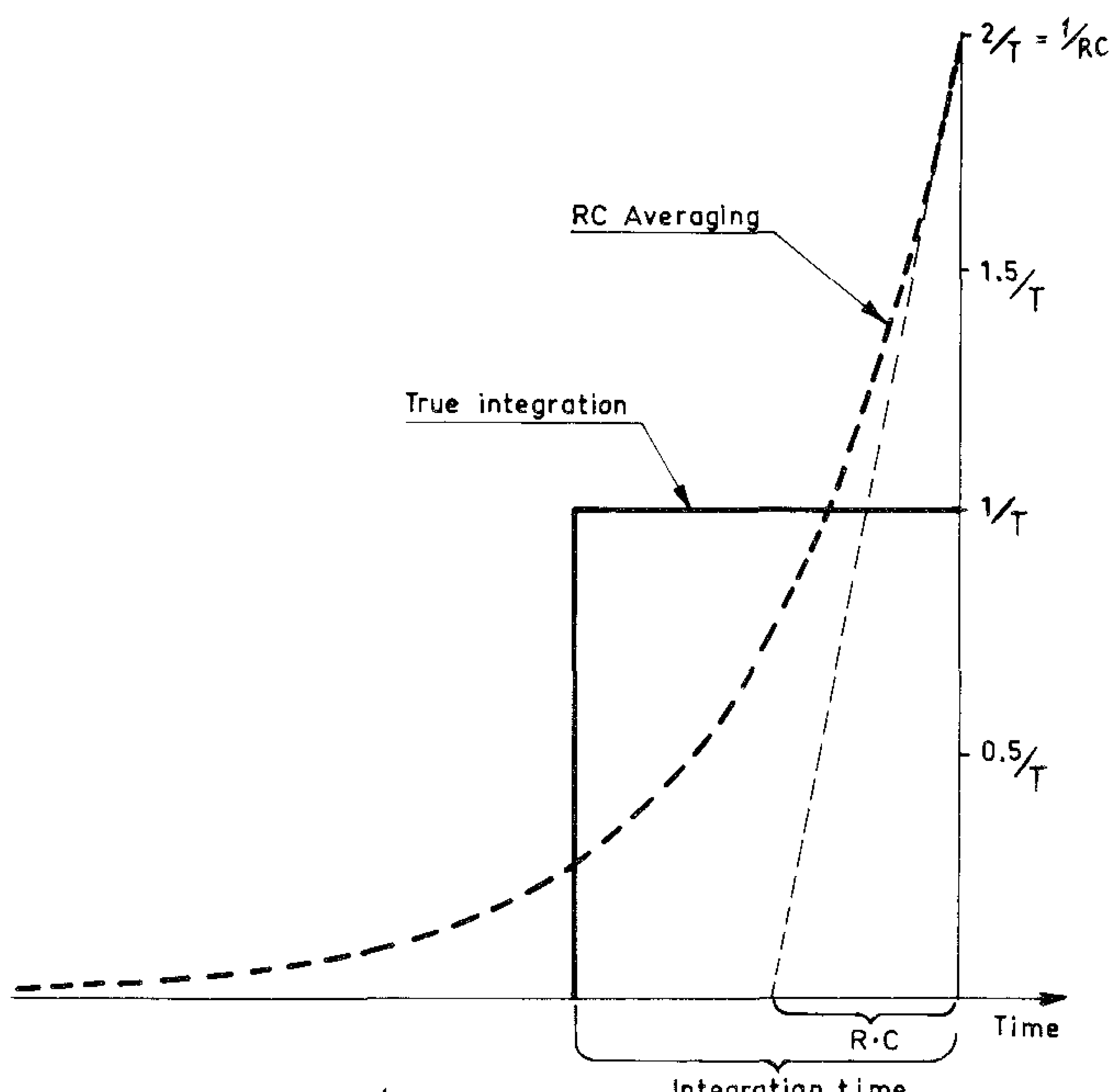

Fig. 3. Weighting curves (Timewise)

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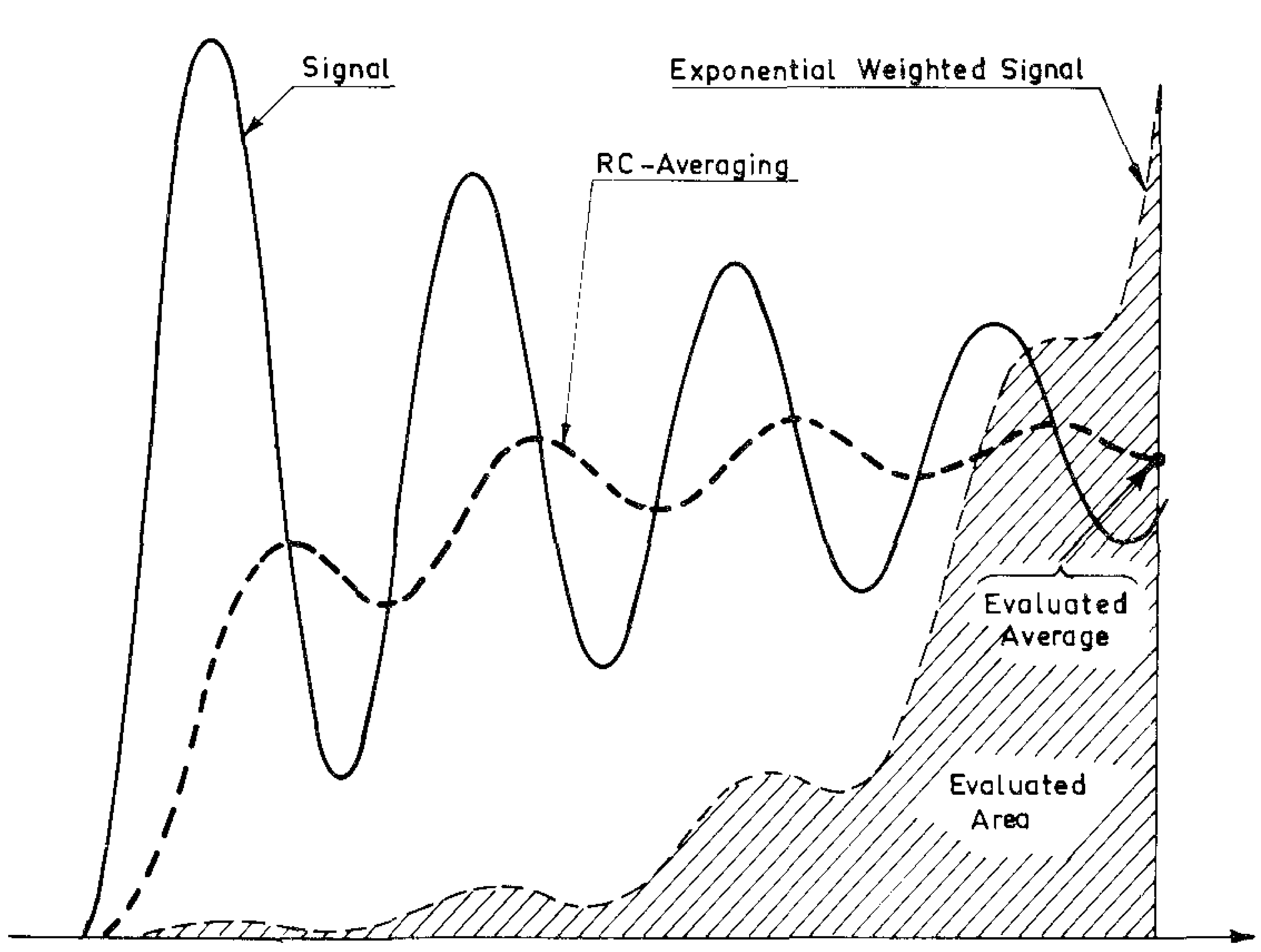

Fig.4. RC–Averaging of a Signal

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Another way of averaging, which is simpler electronically is the RC-low-pass filter. For this system a simple first order differential equation can be written, ref. fig.2. The impulse is a decaying differential equation, ref. fig.2. The impulse response is a decaying exponential. The response to an arbitrary signal is the convolution integral of the signal and the impulse response. This can be considered the same as integrating the signal multiplied with a weighting function, which is the impulse response folded around to negative times, ref. fig.3. In this figure is also shown the time weighting function of true integration, which is 1/T inside the integration interval and zero outside. The scales of the two curves are arbitrary, but they are predisposed to the results of the following by having T equivalent to 2 RC instead of 1 RC. Actually the differences between the two curves are better distributed in this way as the two areas outside the rectangle are only 0.135 and 0.153 whereas the single area outside the rectangle with T = RC would be 0.368.

In fig.4 is shown the result of RC-averaging of the same signal which was used in fig.1.

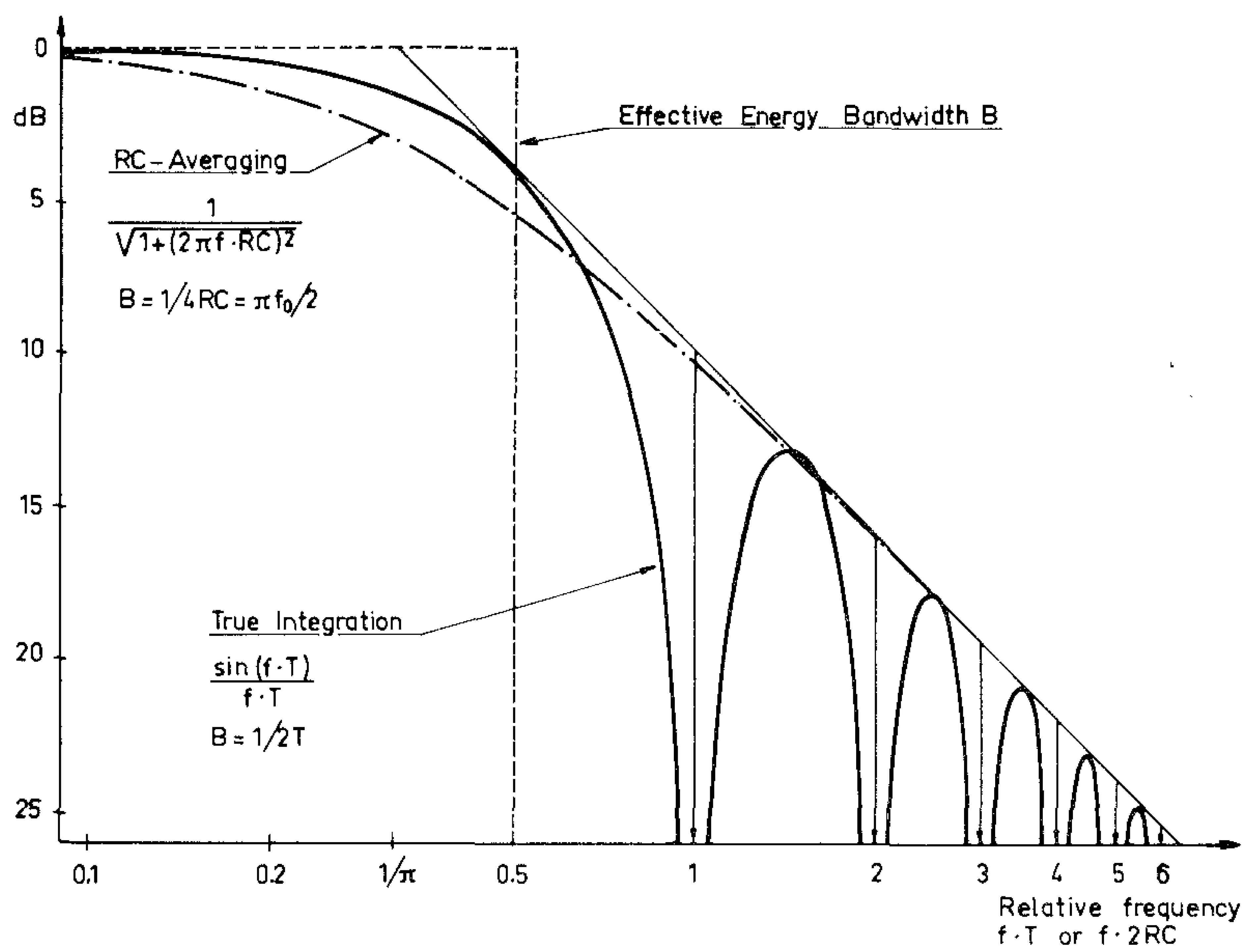

Fig.5. Weighting curves (Frequency – wise).

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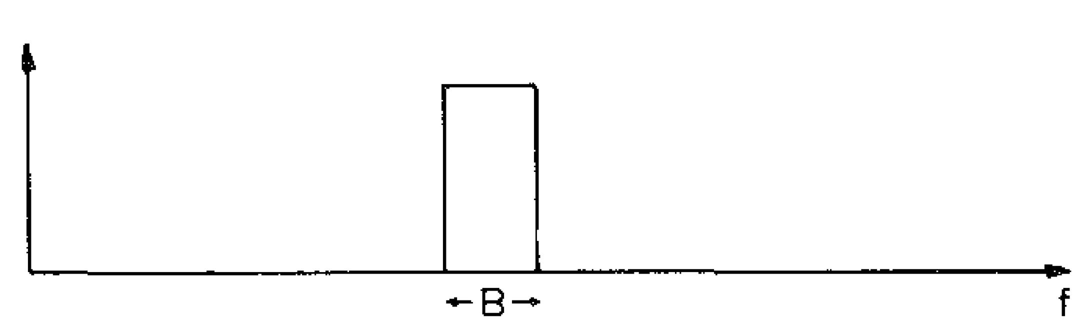
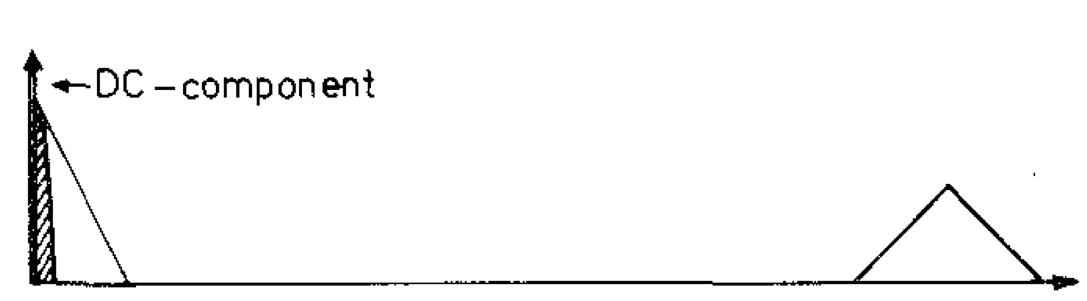
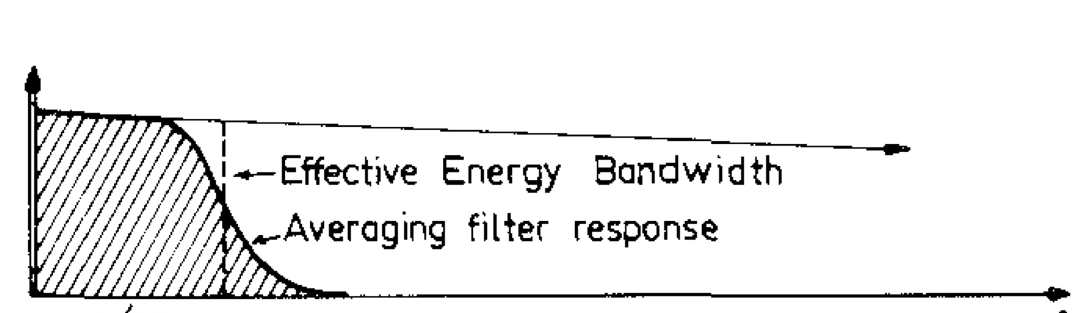
Instead of comparing the two types of averaging on basis of their time weighting curves they can be compared on basis of their frequency weighting curves, ref. fig.5. Again the scales of the two curves are arbitrary, but if T is chosen equal to 2 RC the two curves fit nicely with the same two limiting lines in this double logarithmic plot. However, it is more important that on this basis the two effective power bandwidths are the same. Actually this is the main reason for choosing 2 RC as the equivalent averaging time. If for instance a narrow band of random noise is squared or in other ways rectified the result is a DC plus a low frequence band of random noise, ref. fig.6. If this is filtered by two averaging processes with the same low effective power bandwidth, the standard deviation of the output fluctuations will be the same. (This is further described in lit. ref 1.)



Original narrow band random noise spectrum



Resulting Spectrum after squaring.



Enlarged view of the low frequency part of above spectrum.

Relative standard deviation on RMS :
$\sigma = \frac{1}{2\sqrt{BT}} = \frac{1}{2\sqrt{B\cdot 2RC}}$

Fig. 6. Averaging of squared narrow band random noise.

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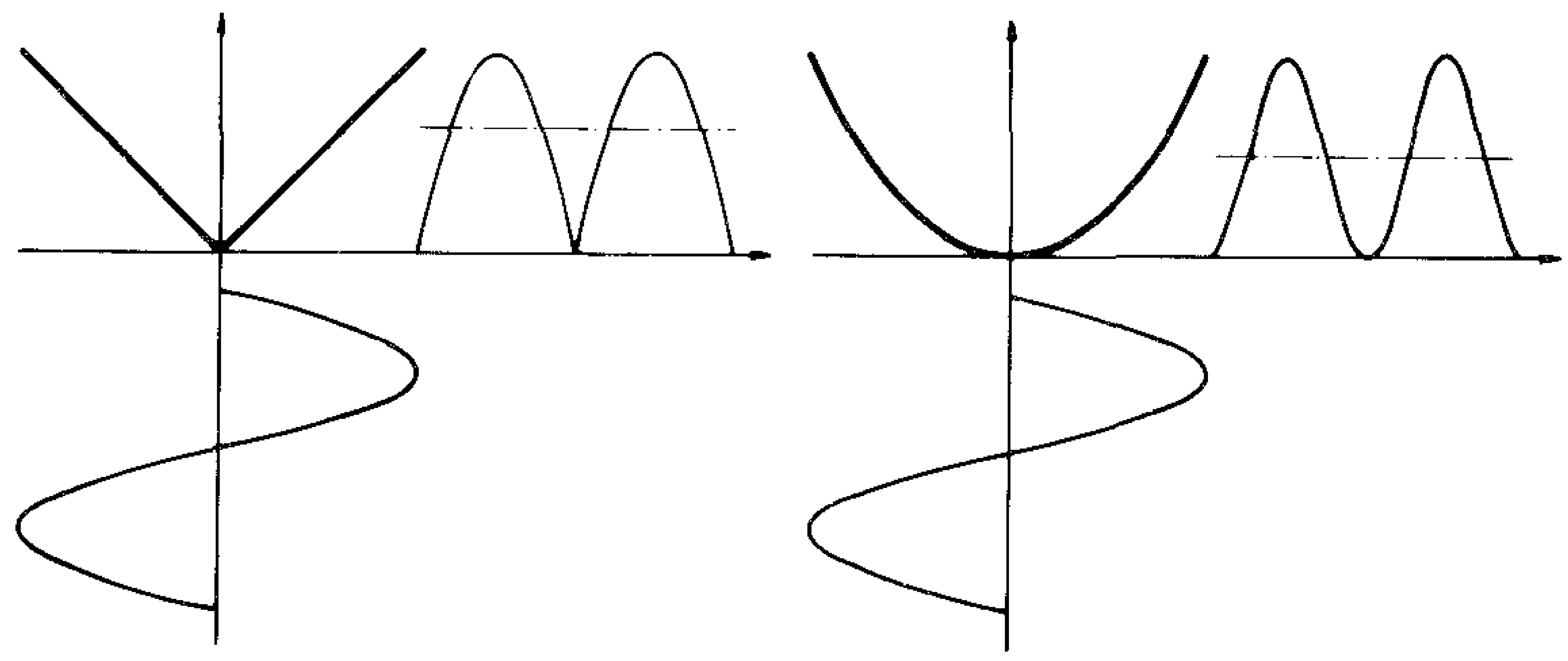
Now, the information wanted from the signal is normally not a simple average. Pure AC signals have zero average independent of the size of the signal. Therefore some amplitude weighting of the instantaneous values is introduced. Electronically the simplest method will be to consider only the values with one sign; or those with negative sign could be turned around thus considering only the absolute or numerical values, ref. fig.7.



Absolute Average   Mean Square

Fig.7. Rectifying Characteristics (Fixed)   171344

Direct Method:

$$RMS = \sqrt{MS}$$

$$U = \sqrt{\overline{V^2}}$$

$$V^2 - U^2 = RC_1 \frac{d(U^2)}{dt}$$

Feedback Method:

$$RMS = \frac{MS}{(RMS)} = M\frac{S}{(RMS)}$$

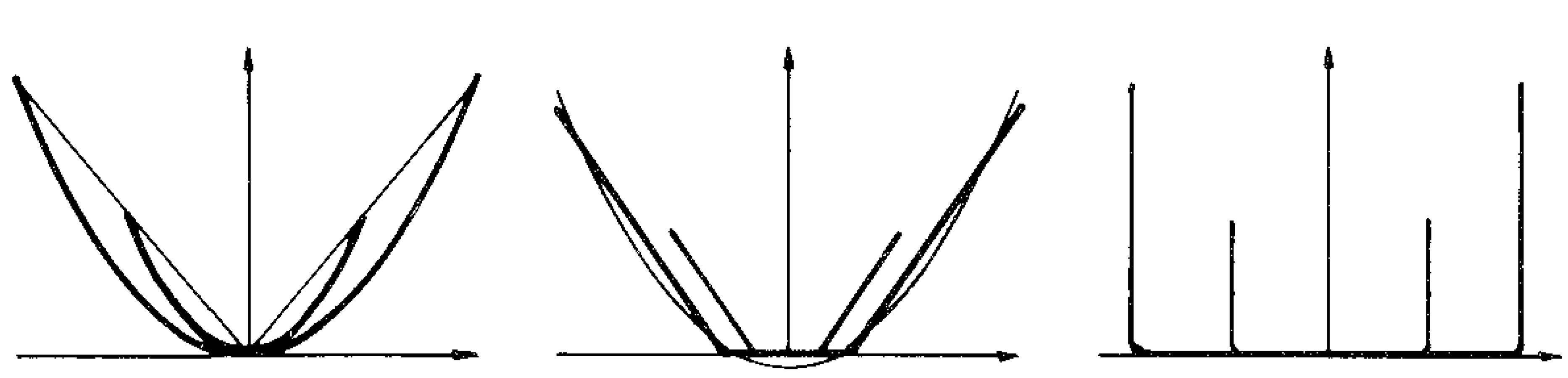
$$U = \frac{\overline{V^2}}{U}$$

$$\frac{V^2}{U} - U = RC_2 \frac{dU}{dt} \quad \text{or}$$

$$V^2 - U^2 = \frac{RC_2}{2} \frac{d(U^2)}{dt}$$

$$T_{av} = 2RC_1 = RC_2$$

Fig.8. Formulas for two methods of obtaining RMS values.   171345



RMS   Quasi RMS   Peak

Fig.9. Rectifying Characteristics (Moveable)   171346

Mathematically this process is not so simple as it looks, because integration has to be split in positive and negative parts. It is much simpler to consider a weighting proportional to the value, that is to use a Mean Square averaging. While the MS value mathematically is more useful, it is more difficult to obtain it electronically. Often a thermal converter is used, as the heating of a current in a resistor is proportional to the MS of the current. Other circuits make a polygonal approximation to the parabolic characteristic of fig.7 by means of a network consisting of diodes and resistors, and still other circuits utilize lin. — log. and log. — lin. conversion to obtain the squaring.

In most cases the Root Mean Square is the value preferred to the MS. It can be obtained by simple square root extraction of the MS value, but it can also be obtained in another way. If the MS is by some means divided by the output signal the same result is obtained, ref. fig.8.

Such a division is for instance done in a polygonal approximating circuit by letting the corner points move proportional to the output voltage, ref. fig.9. Although the MS value can not be found anywhere in the circuit, it will behave exactly as the first type of root mean squaring, as long as the parabola is not overloaded.

Very often sufficient accurate RMS values can be made by a simple polygonal approximation having only two corners as shown in fig.9. Such a circuit may be called a quasi RMS circuit (ref. lit. 2 and 3). If the ratio between the charging and the discharging resistors is approximately 1 : 3, then the error will be less than 1 dB for signals with crest factors up to 3 and signals as sinewaves, squarewaves, triangular waves, and gaussia random noise will show only a few per cent error.

If the ratio between the charging and discharging resistor is decreased to a very low value as shown in fig.9, the circuit turns into a (quasi) peak circuit, the better the smaller the ratio is.

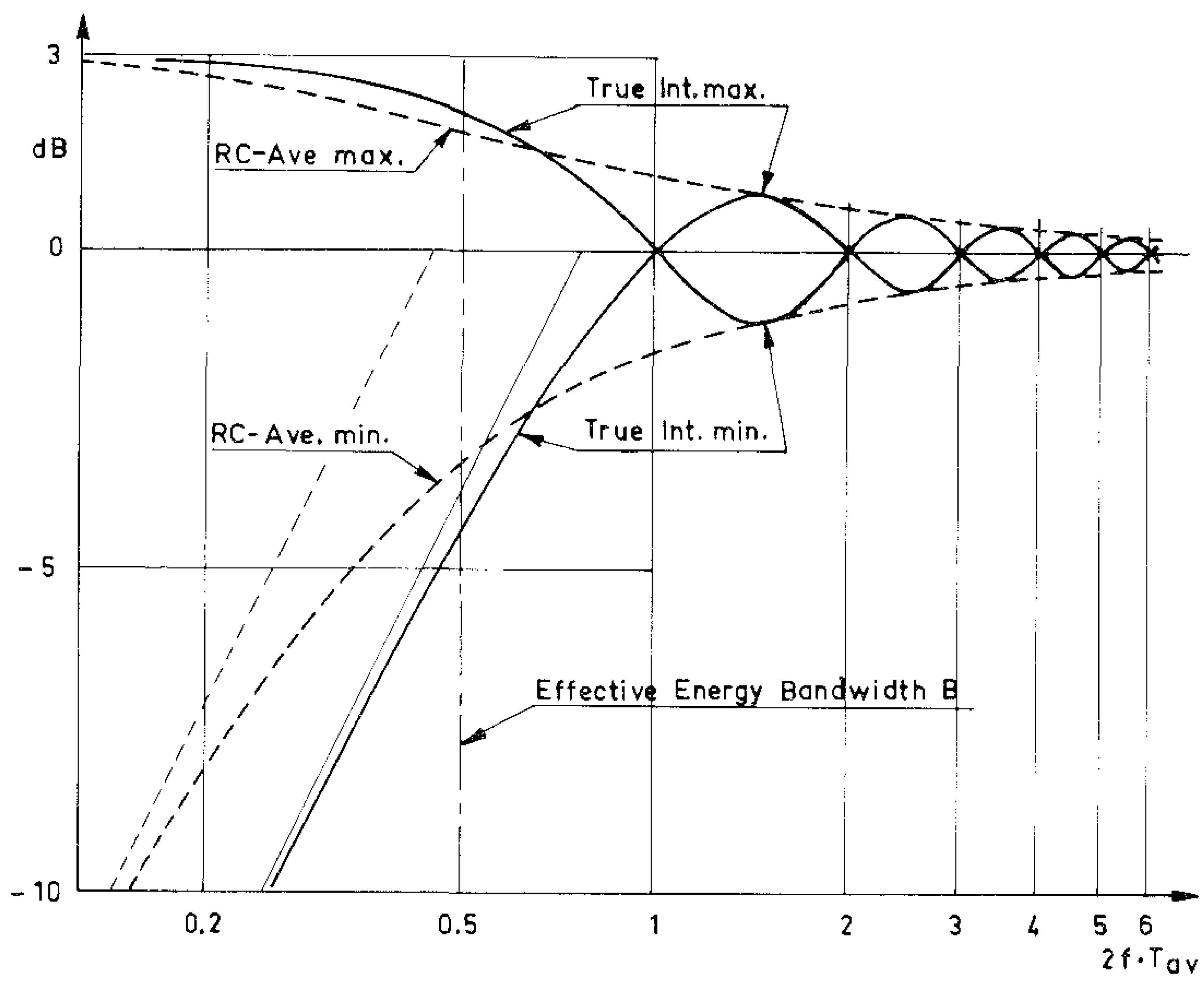


Fig.10. Low frequency rippel for MS and RMS averaging of sinewaves

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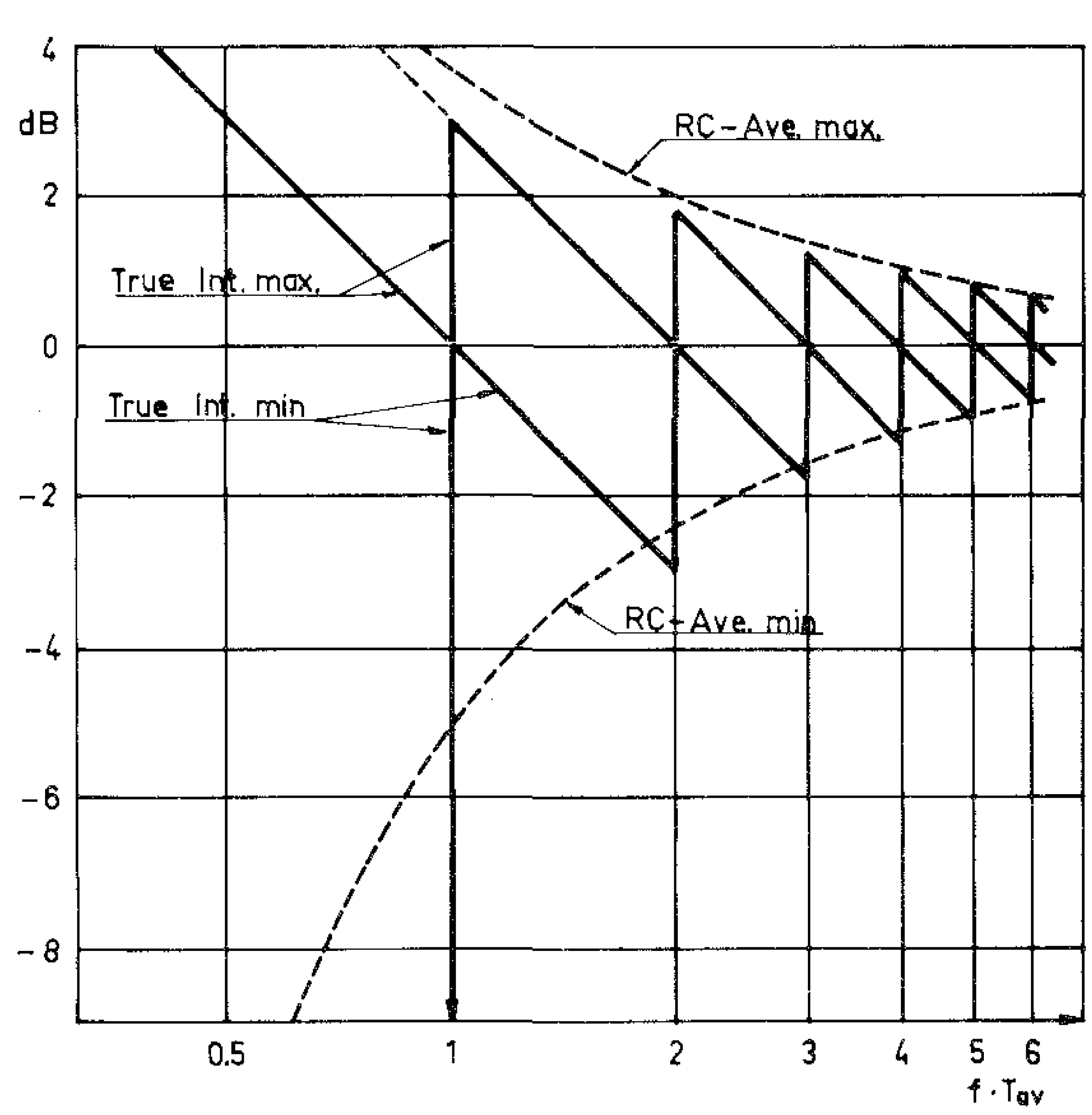

Fig.11. Low frequency rippel for MS and RMS averaging of pulsewaves

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As the MS and RMS is the most interesting values we will start considering these. In fig.5 was shown the frequency responses of the two types of averaging. This will also be the frequency responses versus modulation frequency for a high frequency carrier weakly amplitude modulated by a low frequency sinewave. However, another type of frequency response can be made, namely the ripple obtained when measuring the low frequencies alone. In fig.10 is shown the result for sinewaves while fig.11 shows the result for a wave of narrow pulses. It is seen that in most parts of the figures the ripple for true integration is within the ripple for RC-averaging. This is an argument for choosing a longer timeconstant and especially fig.11 shows that T_{av} = RC is a better choice from this point of view.

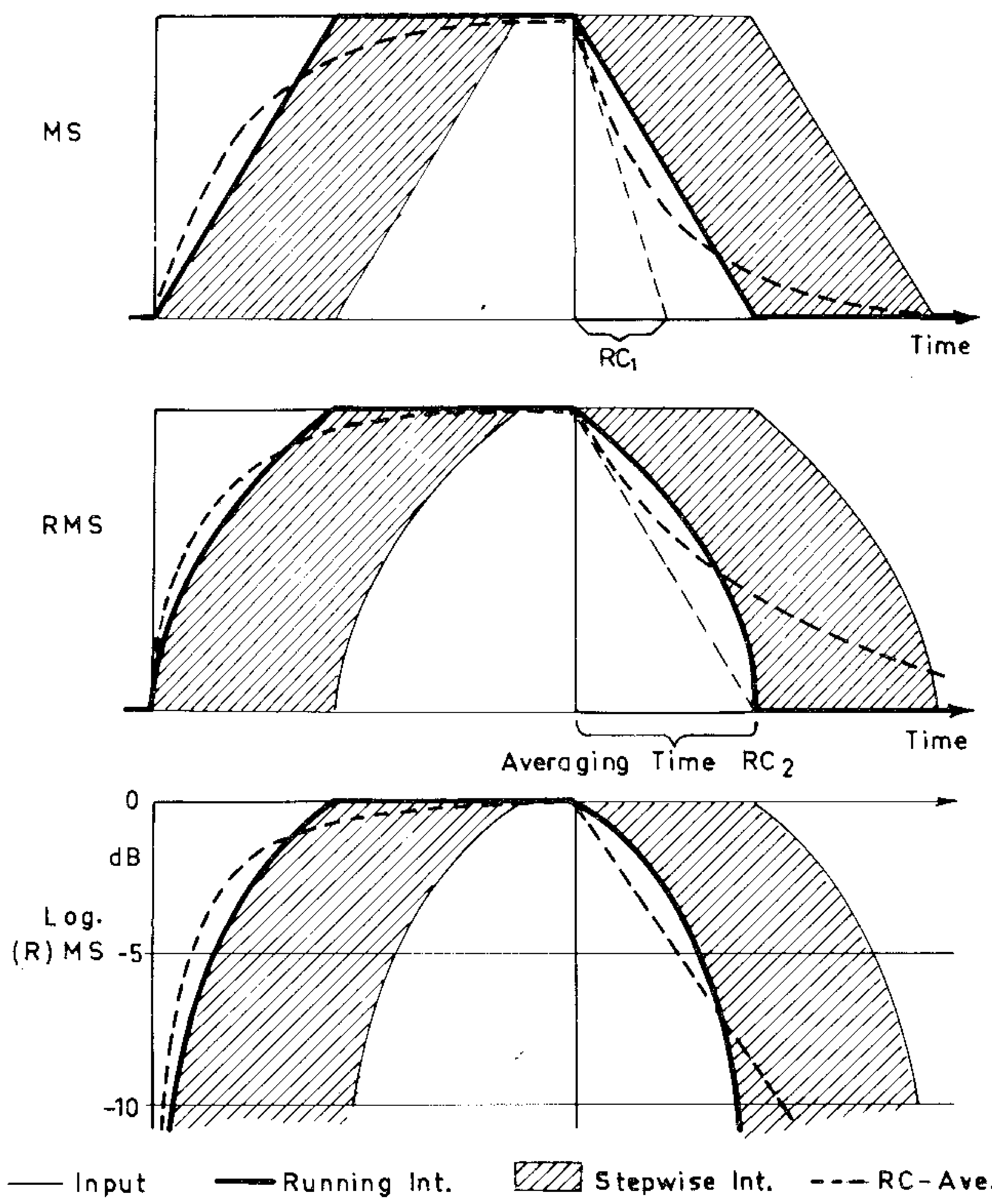

Fig. 12. Step responses of MS and RMS averaging

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Another aspect of interest is the behaviour of the two types of averaging of single rectangular pulses or step functions. In fig.12 are shown first the responses of the mean square. For true integration they are straight sloping lines, whereas for RC-averaging both the rising and the falling response are equal exponential curves with time constant RC_1.

Below the RMS responses. For true integration the rise and fall curves are equal parabolas, but for RC-averaging the two curves are no longer the same. The rising curve is not exponential, while the falling curve is still exponential but having a time constant $RC_2 = 2\,RC_1$. This gives another reason for choosing the $T_{av} = 2\,RC_1 = RC_2$ as the decay time constant can easily be measured whereas in the movable parabola RMS circuit the time constant RC_1 does not exist. (It is due to this confusion with two time constants that the designation "averaging time" has been used in this paper).

Finally is shown the responses with logarithmic ordinates. Here the RC-averaging decay is a straight line, while the rise curve looks even steeper than in the middle figure. In all three figures the choise $T_{av} = 2\,RC_1$ seems to give reasonable equality between true integration and RC-averaging. As mentioned above the moving parabola RMS circuit will behave exactly as the fixed parabola circuit, as long as the parabola is not overloaded. This must, however, always be the case at the beginning of the rise curve, where the parabola is very small. In fig.13 the rise response is therefore shown in double logarithmic coordinates. It is seen that the ideal RMS response is reached approximately, when the response is a factor of C below the final level, where C is the crest factor capability*. That is rectangular pulses of longer duration than about T_{av}/C^2 are integrated as the ideal RMS circuit should do. It is also observable that measuring short single pulses $T_{av} = RC_1 = 1/2\,RC_2$ would be a better choice when comparing true integration with RC-averaging. For such pulses it is, however, normally more important to know the size, width and shape than just the energy of the pulse.

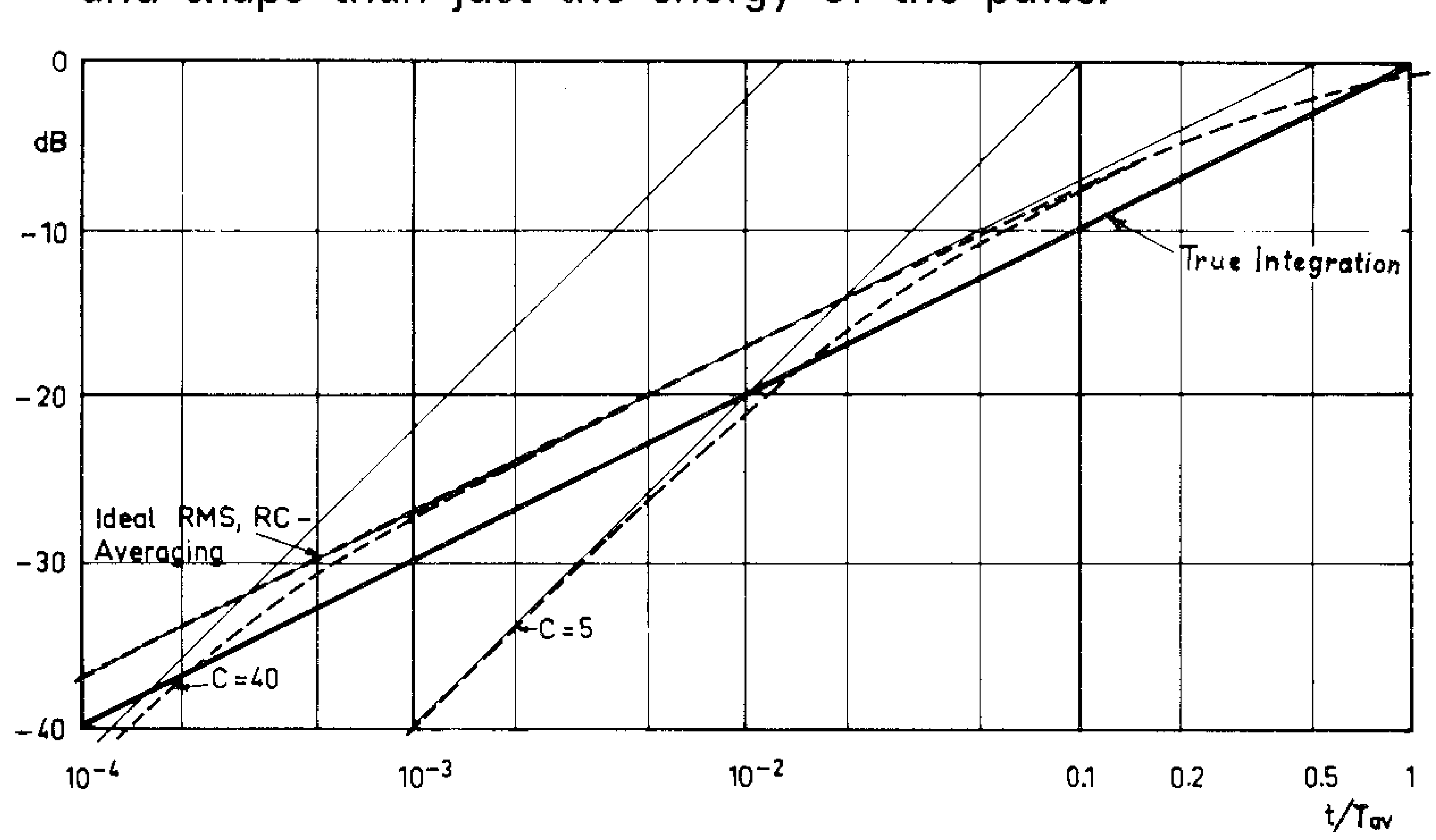

Fig. 13. Rising step response of RMS circuits with limited crest factor capability C.

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Another aspect which is important when comparing averaging processes, is their ability to follow slow changes in signal levels. This is for instance important when measuring frequence responses with a sweeping sinewave.

* In an earlier paper (4) the crest factor capability is defined in a different way corresponding to twice the values used here.

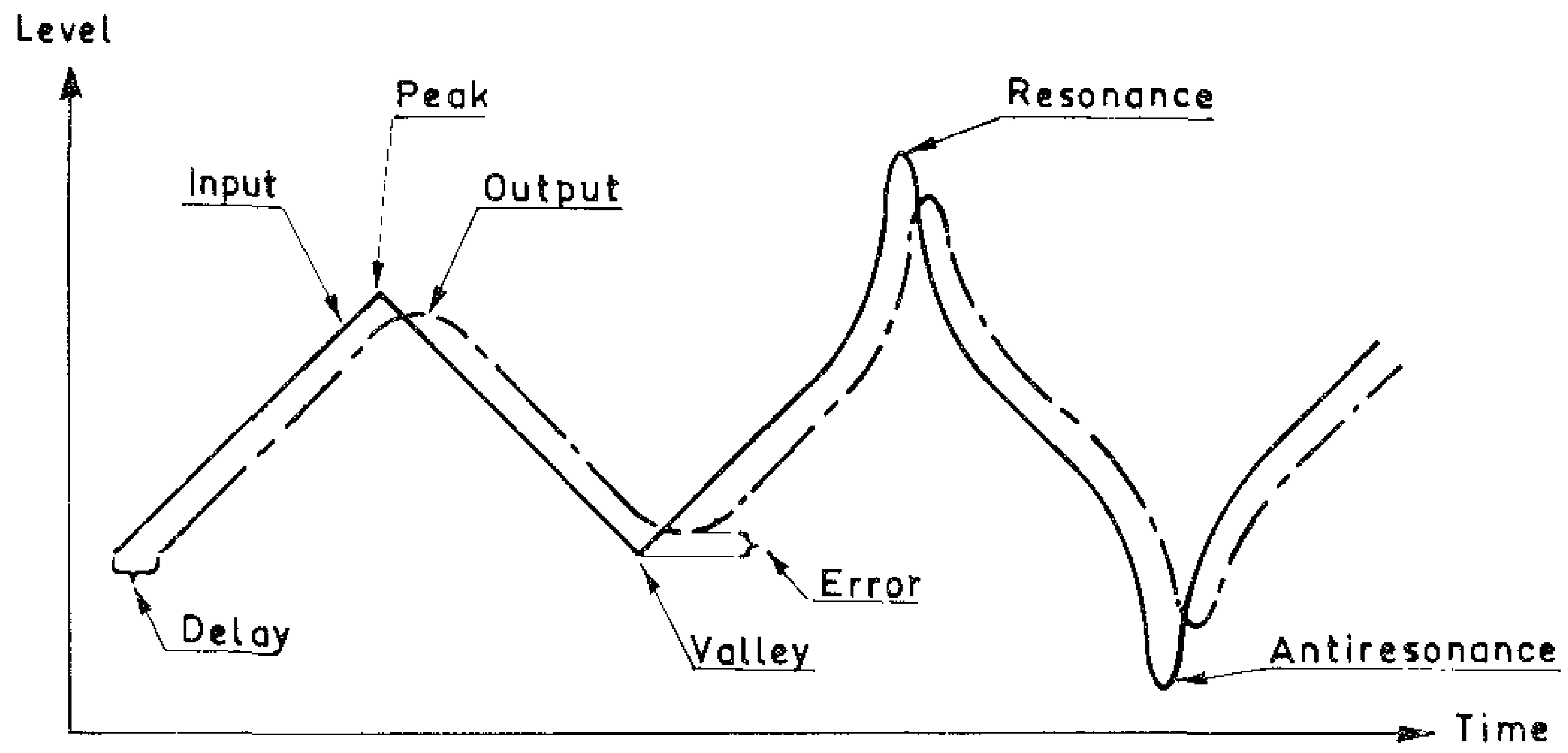

Fig. 14. Response to slow level changes

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As it appears from fig.14 there will be a certain delay which depends on the slope of the curve. The most important parts of a curve like that are normally the peaks and notches, and here the averaging also produces errors in the levels. Normally these peaks and notches are not sharp corners as shown on the first part of the curve, but round resonances, which can be treated as parabolas near their tops respectively bottoms.

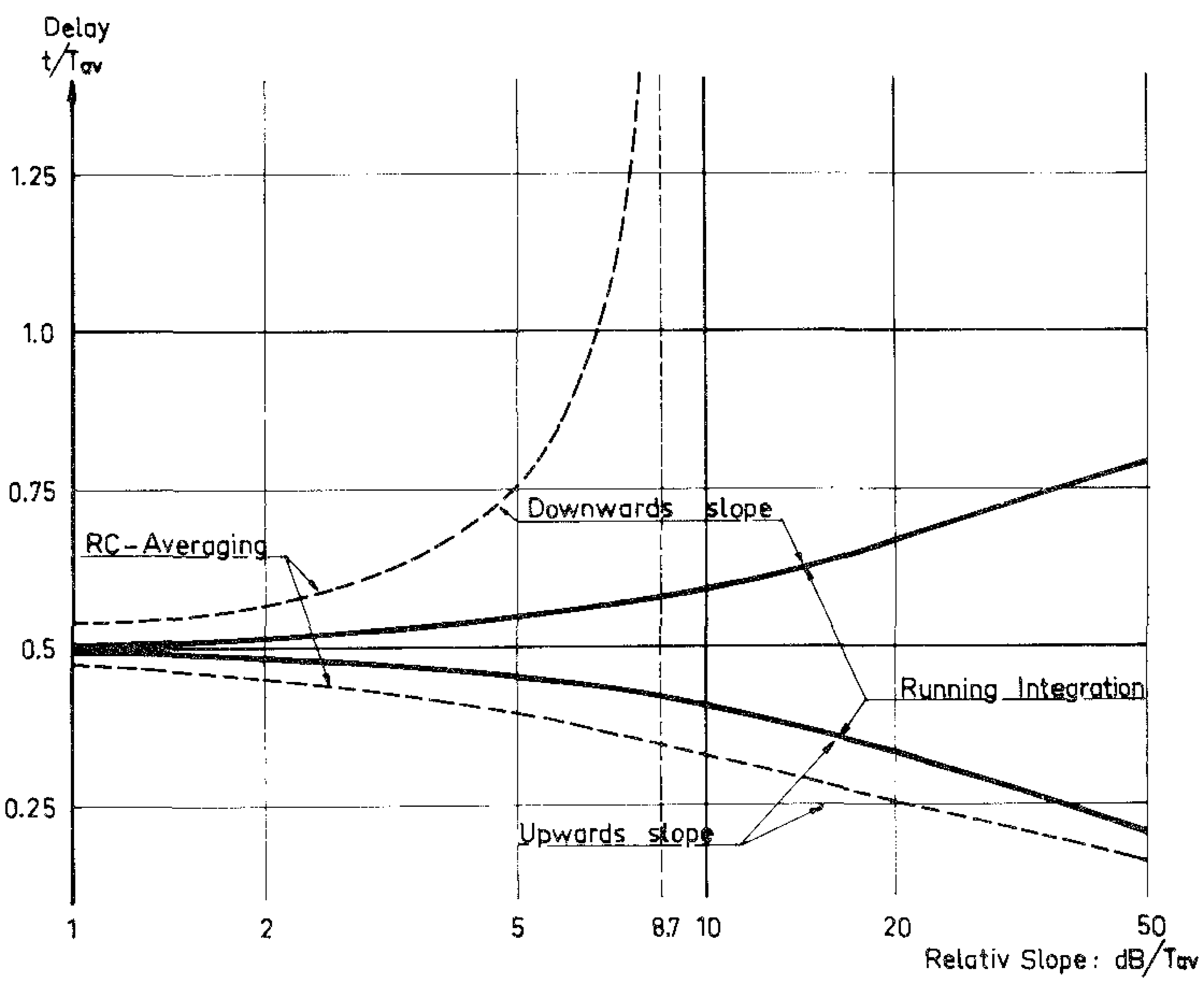

Fig. 15. Delay for slow changes. MS and RMS.

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In fig.15 the resulting delay is shown as function of the slope. It appears that the delay is approximately 1/2 T_{av} for low slopes for both running integration and RC-averaging. This is another strong argument for choosing $T_{av} = 2\ RC_1 = RC_2$. It also appears that the RC averaging is faster in the rising and slower in the falling than the running integration, and that it will never be able to follow a downward slope of more than 8.7 dB/T_{av}.

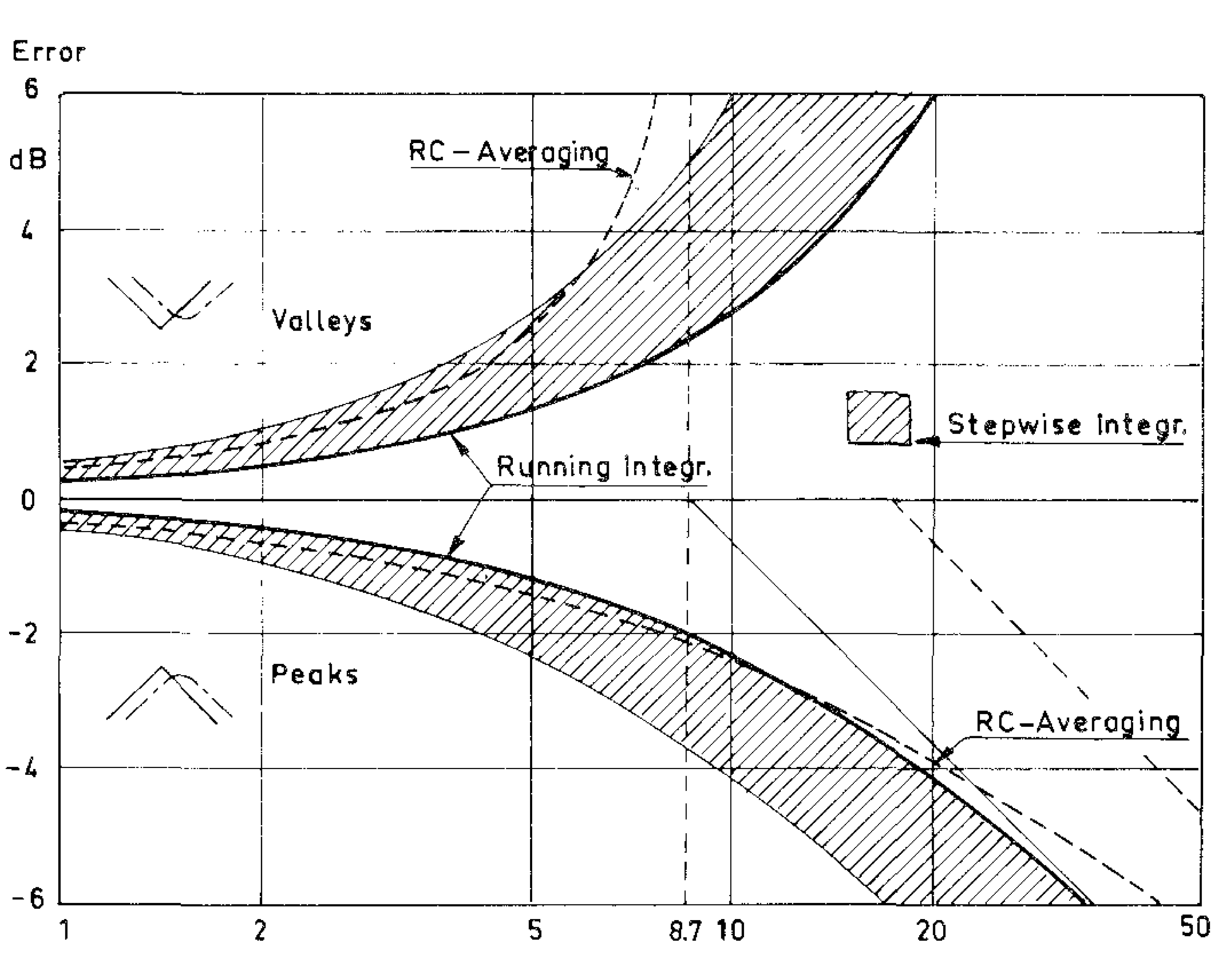

Fig. 16. Errors on Peaks and Valleys. MS and RMS

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Fig.16 shows the errors on the sharp peaks and valleys as a function of the slopes of the sides. The curves are based on the assumption that the leading slope has had sufficient duration to obtain the delays shown in the previous figure. Again we see that RC-averaging cannot follow down into a valley with slopes above 8.7 dB/T_{av}. However, even for slopes up to $5 - 6$ dB/T_{av} RC-averaging is better than stepwise integration in the worst case, and for the peaks with high slopes it is even better than running integration.

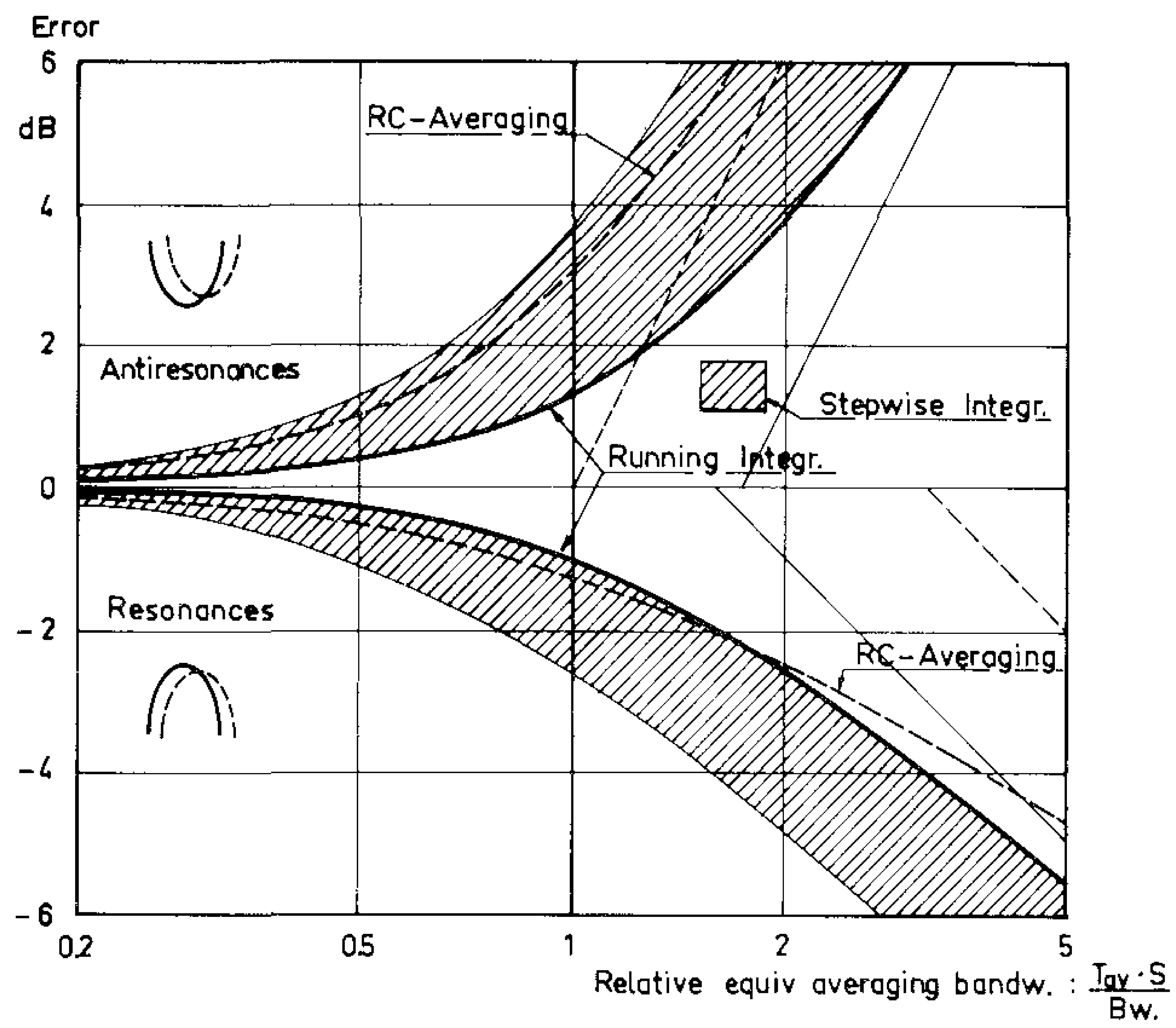

Fig. 17. Errors on Resonances and Antiresonances. MS and RMS.

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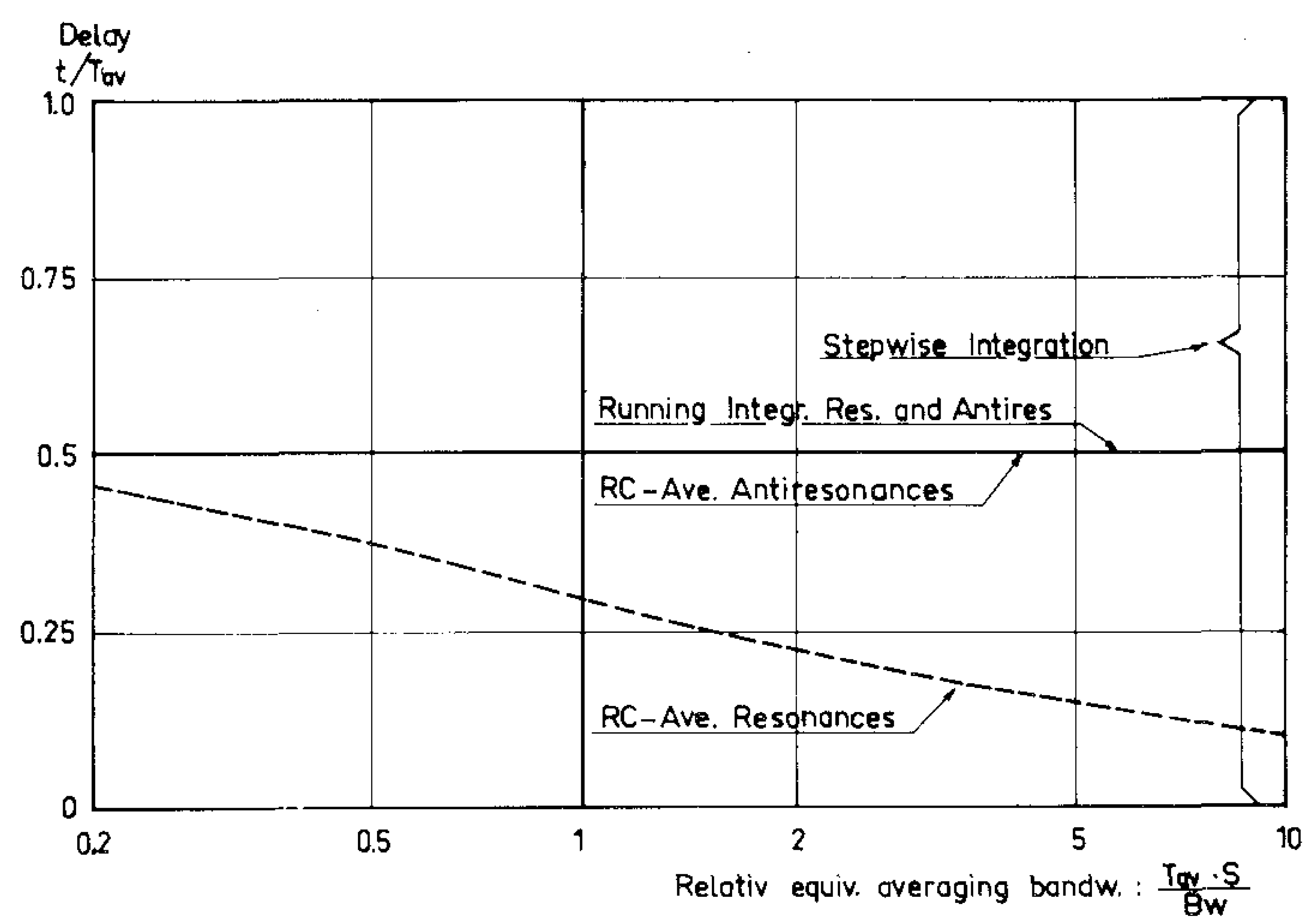

Fig. 18. Delays of Resonances and Antiresonances. MS and RMS.

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Fig.17 shows the errors on resonances and antiresonances as function of the relative equivalent averaging bandwidth: $T_{av} \cdot S/Bw$ where S is the sweep speed and Bw is the 3 dB bandwidth. The shape of the antiresonances is parabolic and not exponential as the valleys used in fig.16 and therefore there is no vertical asymptote on the curve for RC-averaging. Again it is seen that the RC-averaging fits within the shaded area of stepwise integration and is even better than running integration for resonances at the right end of the curve.

As the delays of the peaks and notches may also have some importance, these are shown in fig.18. The running integration here has an advantage to the others, as the delay is constant and therefore easier to correct for.

We may conclude that for MS and RMS measurements true integration and RC-averaging are relatively equal in most cases if the averaging time is set equal to the RMS decay time constant RC_2 $(= 2\,RC_1)$ except of impulsive signals or short single pulses where the comparison is best if $T = 1/2\ RC_2$.

It is the intention in a future Brüel & Kjær Technical Review to give the full article covering also the results on absolute averaging, quasi RMS and peak measurements as well as the derivation of the different curves.

References:

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Printed in Denmark B & K Offset